SWOT INSTITUTE
APPLICATION OF DERIVATIVES
XII-TEST

Time : 1 hr.

1. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.
2. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.
3. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.
4. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$.
5. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.
6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is
   (a) parallel to the line $2x - y + 9 = 0$
   (b) perpendicular to the line $5y - 15x = 13$.
7. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.
8. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles* if $8k^2 = 1$.
9. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.
10. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.
11. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) strictly increasing (b) strictly decreasing.
12. Find intervals in which the function given by $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is (a) increasing (b) decreasing.
13. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing ?
14. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When x = 10 cm and y = 6 cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.